7.5a Applications from Science and Statistics

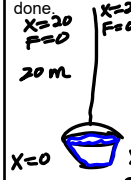
Work done by a variable force: $W = \int_a^b F(x)\,dx$ Force can vary

Work = Force · distance { Force is constant }

Dec 17-7:20 PM

A leaky bucket weighs 22N empty. It is lifted from the ground at a constant rate at a point 20m above the ground by a rope weighing 0.4 N/m. The bucket starts with 70N of water but it leaks at a constant rate and just finishes draining as the bucket reaches the top. Find the amount of work done.

F linear



bucket: $W = 22\text{ N} \cdot 20\text{ m} = 440\text{ Nm} = 440\text{ J}$

water slope $= \frac{70-0}{0-20} = -3.5\ \frac{N}{m}$

$F = -3.5(x-0)+70 = -3.5x+70$

$W = \int_0^{20} -3.5x+70\,dx = 700\text{ J}$

rope $F = -.4(x-0)+8 = -.4x+8$

$W = \int_0^{20} -.4x+8\,dx = 80\text{ J}$

$+$

1220 J

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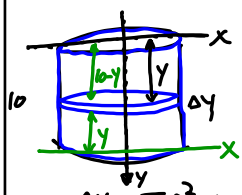
How much work does it take to pump all the water over the rim of a cylindrical tank of height 10ft and diameter 10ft? $62.4\ \frac{lb}{ft^3}$



$\Delta V = \pi r^2 \Delta y$ ($r = 5$)

volume of slice

$W = \int_0^{10} 62.4 \cdot \pi \cdot 5^2 (10-y)\,dy$

$= 245{,}044$

ΔW = work for one slice

ΔW = Force · distance

ΔW = weight · y

$= 62.4\ \frac{lb}{ft} \cdot \Delta V\,ft \cdot y\,ft$

$= 62.4 \cdot \pi \cdot 5^2 \cdot \Delta y \cdot y$

$\Delta W = 62.4\,\pi \cdot 5^2 \cdot y \cdot \Delta y$

$W = \int_0^{10} 62.4 \cdot \pi \cdot 5^2 \cdot y\,dy$

$W = 245{,}044\text{ ft·lb}$

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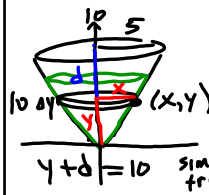
A conical tank of height and diameter 10ft is filled to within 2 ft of the top with olive oil weighing 57 lb/ft^3. How much work does it take to pump the oil to the rim of the tank?



ΔW = Force · distance

= weight · distance

$= 57 \cdot \Delta V \cdot (10-y)$

$= 57 \cdot \pi r^2 h\,(10-y)$

$= 57\,\pi x^2 \Delta y\,(10-y)$

$\Delta W = 57\,\pi \left(\frac{1}{2}y\right)^2 (10-y)\,\Delta y$

$W = \int_0^8 57\,\pi \left(\frac{1}{2}y\right)^2 (10-y)\,dy$

$= 30{,}561\text{ ft-lb}$

similar triangles $\frac{x}{y} = \frac{5}{10}$

$d = 10-y$

$h = \Delta y$

$r = x$

$x = \frac{1}{2}y$

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Dec 13-10:10 AM